GCSE Maths – Geometry and Measures

Volume of 3D Shapes

Worksheet

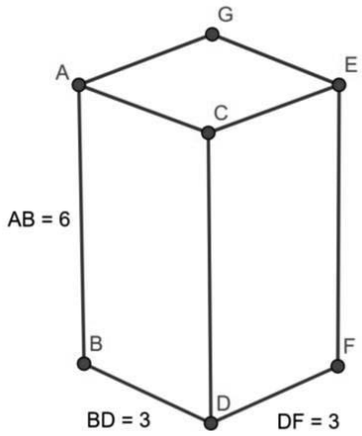
WORKED SOLUTIONS

This worksheet will show you how to work out different types of volume of 3D shapes questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the volume of the cuboid shown below.



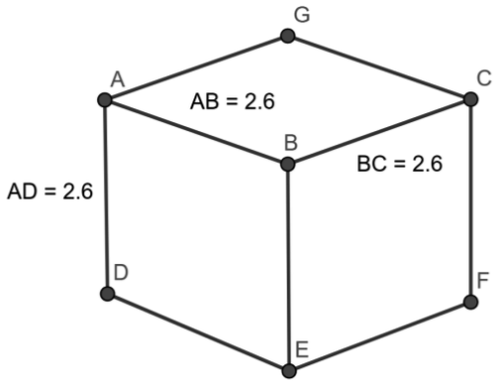
Step 1: For cuboids, use the formula: $Volume = length \times width \times height$.

We have been given the length, width, and height, so we simply multiply these together. Remember to use the correct units!

$$Volume = 6 \times 3 \times 3 = \mathbf{54\ units^3}$$

Guided Example

Find the volume of this cube.



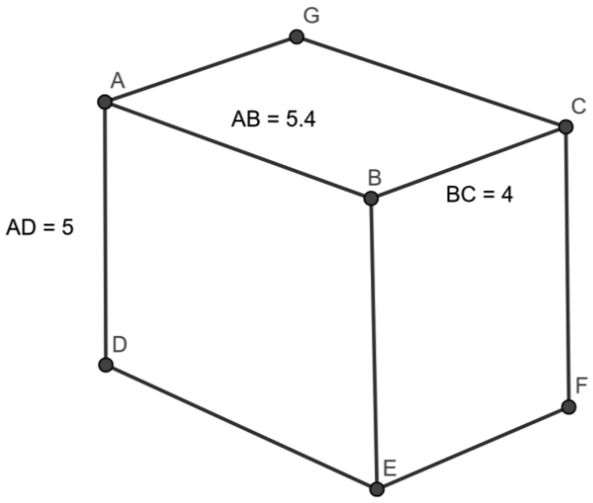
Step 1: For cuboids, use the formula: $Volume = length \times width \times height$.

$2.6 \times 2.6 \times 2.6 =$ 17.576 units^3

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

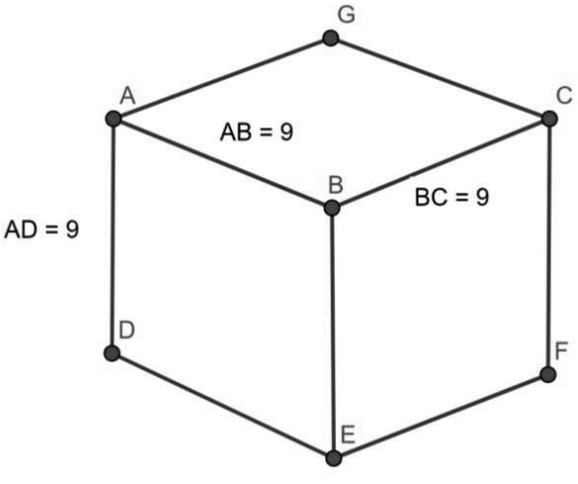
1. Calculate the volume of the following cubes and cuboids:

a)



Volume = l x w x h
= 5.4 x 4 x 5
= 108 unit^3

b)

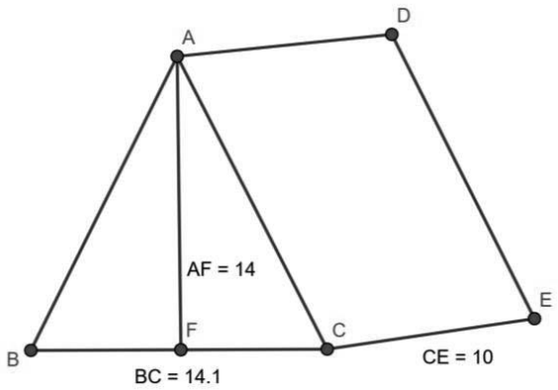


Volume = l x w x h
= 9 x 9 x 9
= 729 units^3

Section B

Worked Example

Find the volume of this prism.



Step 1: When finding the volume of prisms and cylinders, the approach is the same: first, find the cross-sectional area.

As the cross-section here is a triangle, we use the formula for the area of a triangle:

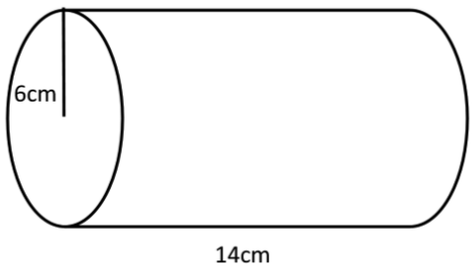
$$Area = \frac{Base \times Height}{2} = \frac{14.1 \times 14}{2} = 98.7 \text{ units}^2$$

Step 2: Multiply the cross-sectional area by the length of the prism.

$$Volume = 98.7 \times 10 = \mathbf{987 \text{ units}^3}$$

Guided Example

Find the volume of the cylinder.



Step 1: When finding the volume of prisms and cylinders, the approach is the same: first, find the cross-sectional area.

Area of circle = $\pi \times 6^2$

$= 36\pi$ units^2

Step 2: Multiply the cross-sectional area by the length of the cylinder.

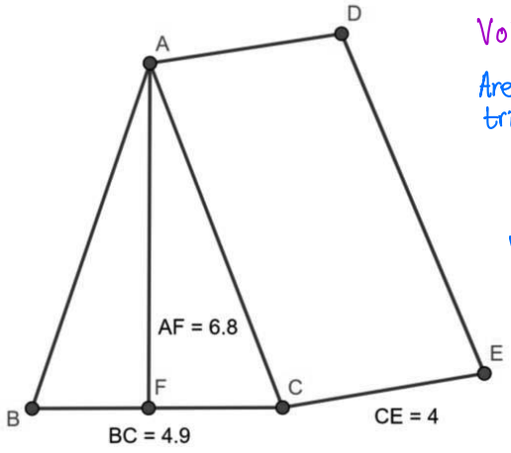
$36\pi \times 14 = 504\pi$ units^3

or 1580 units^3 (3sf)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Calculate the following:

a) The volume of this prism



Volume = cross-sectional area × depth

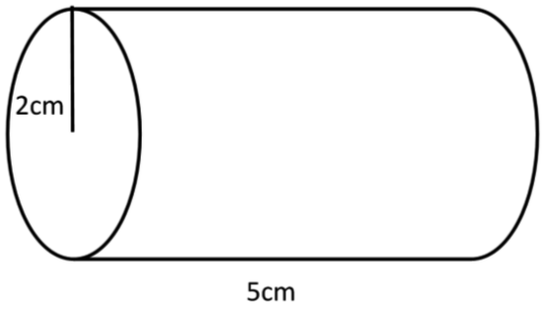
Area of triangle $= \frac{1}{2} \times 6.8 \times 4.9$

$= 16.66 \text{ units}^2$

Volume $= 16.66 \times 4$

$= 66.64 \text{ units}^3$

b) The volume of this cylinder



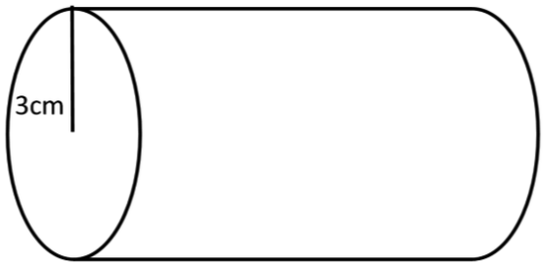
Volume = cross-sectional area × depth

Area of Circle $= \pi \times 2^2 = 4\pi \text{ cm}^2$

Volume $= 4\pi \times 5$

$= 20\pi \text{ cm}^3$ or 62.8 cm^3 (3sf)

c) The length of this cylinder if the volume is 226.19 cm^3



Volume = cross-sectional area × depth

Area $= \pi \times 3^2 = 9\pi \text{ cm}^2$

$226.19 = 9\pi \times \text{length}$

$\div 9\pi$

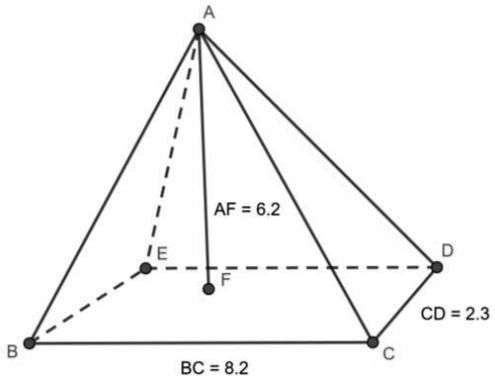
$8 = \text{length}$

length = 8cm

Section C

Worked Example

Find the volume of this pyramid.



Step 1: Find the area of the base.

This is a rectangle-based pyramid. To find the area of the rectangle, we multiply the length by the width.

$$Area\ of\ base = 8.2 \times 2.3 = 18.86\ \text{units}^2$$

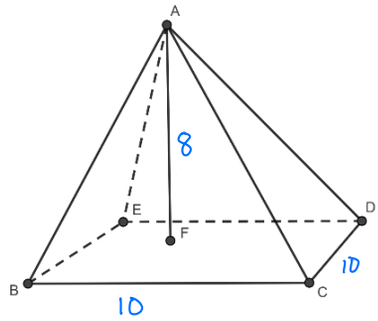
Step 2: Use the formula for the volume of a pyramid.

$$Volume = \frac{1}{3} \times Perpendicular\ height \times Area\ of\ base$$

$$Volume = \frac{1}{3} \times 6.2 \times 18.86 = \mathbf{38.98\ units^3}$$

Guided Example

Given that $\mathbf{BC = CD = 10\ cm}$ and $\mathbf{AF = 8}$, find the volume of the pyramid.



Step 1: Find the area of the base.

$10 \times 10 = 100\ \text{units}^2$

Step 2: Use the formula for the volume of a pyramid.

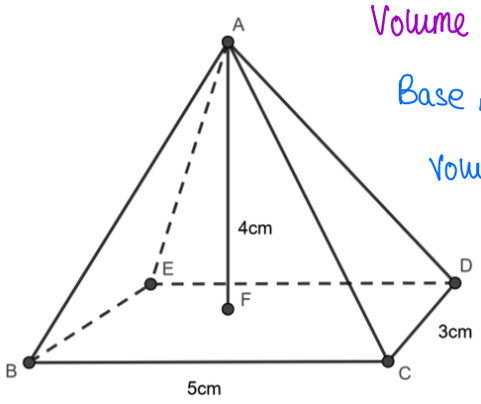
$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$

$= \frac{1}{3} \times 100 \times 8 = \frac{800}{3}\ \text{units}^3$ or $267\ \text{units}^3$ (3sf)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Calculate the following:

a) The volume of this pyramid



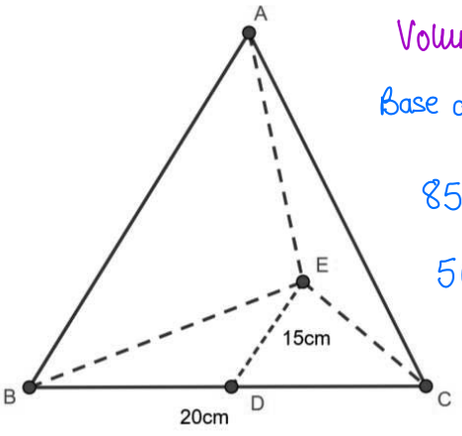
Volume = $\frac{1}{3}$ × base area × height

Base Area = $5 \times 3 = 15\text{cm}^2$

Volume = $\frac{1}{3} \times 15 \times 4$

= 20cm^3

b) The perpendicular height of this triangle-based pyramid if its volume is 850 cm^3



Volume = $\frac{1}{3}$ × base area × height

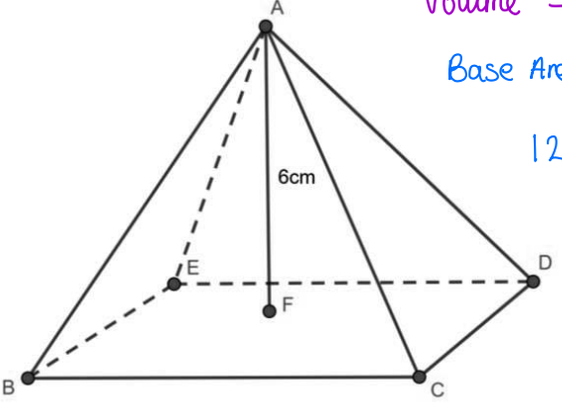
Base area = $\frac{1}{2} \times 15 \times 20 = 150\text{cm}^2$

$850 = \frac{1}{3} \times 150 \times \text{height}$

$50 \times \text{height} = 850$ ÷50

height = 17cm

c) The length of the one of the sides of this square-based pyramid, if the total volume is 128 cm^3



Volume = $\frac{1}{3}$ × base area × height

Base Area = $l \times l = l^2$ ← square

$128 = \frac{1}{3} \times l^2 \times 6$

$2l^2 = 128$ ÷2

$l^2 = 64$

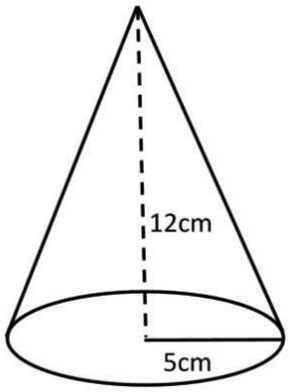
$l = 8\text{ cm}$

lengths can't hold a negative value

Section D

Worked Example

Find the volume of the cone.



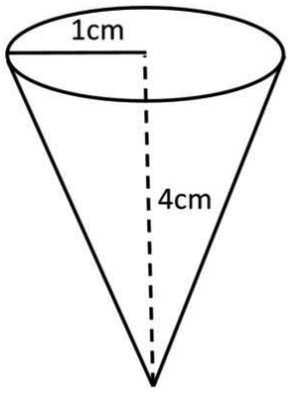
Step 1: Use the formula for the volume of a cone: $Volume = \frac{1}{3}\pi \times r^2 \times h$

Substitute the values we know into the formula to find the volume:

$$Volume = \frac{1}{3} \times \pi \times 5^2 \times 12 = \mathbf{314.16\ cm^3}$$

Guided Example

Find the volume of the cone.



Step 1: Use the formula for the volume of a cone: $Volume = \frac{1}{3}\pi \times r^2 \times h$

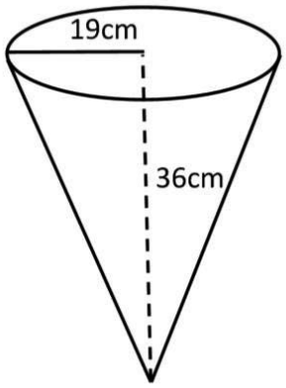
$Vol = \frac{1}{3} \times \pi \times 1^2 \times 4$

$= \frac{4}{3}\pi\ cm^3$ or $4.19\ cm^3$ (3sf)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Calculate the following:

a) The volume of this cone

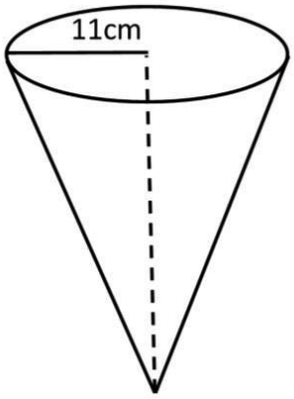


$\text{Volume} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (19)^2 \times 36$

$= 4332\pi \text{ cm}^3$ OR 13600 cm^3 (3sf)

b) The height of this cone if its volume is 2534.22 cm^3



$\text{Volume} = \frac{1}{3}\pi r^2 h$

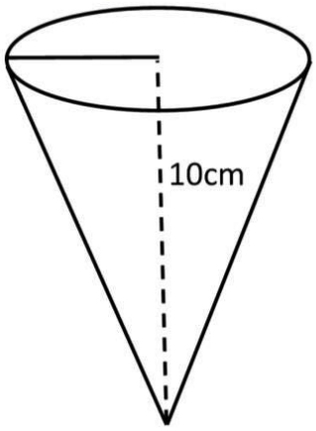
$2534.22 = \frac{1}{3}\pi \times 11^2 \times h$

$\frac{121}{3}\pi h = 2534.22$

$\div \frac{121}{3}\pi$

$h = 20\text{cm}$

c) The radius of this cone if its volume is 167.55 cm^3



$\text{Volume} = \frac{1}{3}\pi r^2 h$

$167.55 = \frac{1}{3}\pi r^2 \times 10$

$\frac{10}{3}\pi r^2 = 167.55$

$\div \frac{10}{3}\pi$

$r^2 = 16$

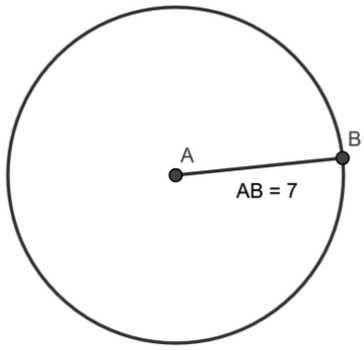
$\sqrt{\ }$

$r = 4\text{cm}$ ← radius can only be positive

Section E

Worked Example

Find the volume of the sphere, given that point A is centred at the origin.



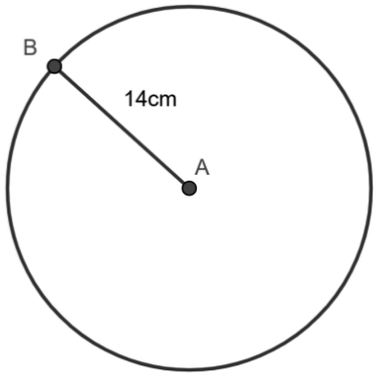
Step 1: Use the formula for the volume of a sphere: $Volume = \frac{4}{3} \times \pi \times r^3$

Substitute the values we know into the formula to find the volume:

$$Volume = \frac{4}{3} \times \pi \times 7^3 = \mathbf{1436.76\ units^3}$$

Guided Example

Find the volume of the sphere, given that point A is centred at the origin.



Step 1: Use the formula for the volume of a sphere: $Volume = \frac{4}{3} \times \pi \times r^3$

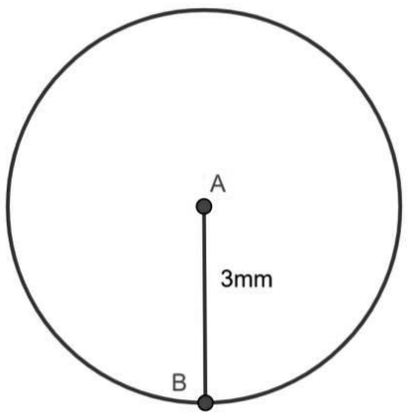
Volume = $\frac{4}{3} \pi \times 14^3$

= 11,494 cm^3

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Calculate the following:

a) The volume of this sphere

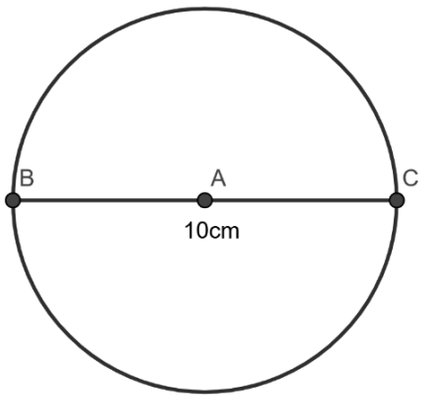


$\text{Volume} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3^3$

$= 36\pi \text{ mm}^3$ OR 113 mm^3 (3sf)

b) The volume of this sphere



$\text{Volume} = \frac{4}{3}\pi r^3$

Radius $= 10 \div 2 = 5 \text{ cm}$

$\text{Volume} = \frac{4}{3}\pi \times 5^3$

$= 130.899...$

$= 131 \text{ cm}^3$ (3sf)

6. A sphere has volume 3053.63 cm^3. Calculate the radius of the sphere.

$\text{Volume} = \frac{4}{3}\pi r^3$

$3053.63 = \frac{4}{3}\pi r^3$

$\div \frac{4\pi}{3}$

$r^3 = 729$

$\sqrt[3]{\ }$

$r = 9 \text{ cm}$